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MHD mixed convective and chemical reactive Couple Stress fluid through expanding or contracting porous pipe in presence of thermal radiation

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--ABSTRACT---
The purpose of this paper, the thermal radiation on an unsteady incompressible laminar MHD mixed convective flow of couple stress fluid through a uniformly expanding or contracting porous pipe which time-dependent rate in presence chemical reaction is to examine. The constitutive relation which governing the flow, energy, concentration and with associate boundary conditions are transformed in to nonlinear coupled system of ordinary differential equations by using similar variables. HAM based Mathematica sofware package BVPh2.0 has been employed to obtain the solution. The influence of different emerging non-dimensional important parameters on velocities, temperature and concentration profiles analyzed and exhibited in the form of graphs. The temperature of the fluid is increased by increasing of inverse Darcy parameter whereas the velocity is decreased. The temperature and concentration are increased with chemical reaction parameter.
KEYWORDS;- *MHD, couple stress fluid, porous pipe, BVPh 2.0, mixed convection, thermal radiation.*

I. INTRODUCTION

The studies referring to unsteady laminar flow through expanding / contracting porous pipe have great attention because of its enormous applications in engineering and biological models such as, binary gas diffusion, food preservation, regression of the burning surface in solid rocket motors, cardiovascular pumping, the model of pulsating diaphragms, blood circulation in the respiratory system, filtration, blood flow and artificial dialysis.[7b, 12]. Terrill and Thomas [1] have been investigated a steady viscous fluid flow through a uniform porous pipe with constant suction / injection at the wall. Quaile and Levy [2] have considered a fluid flow through a circular porous tube cavity with suction throughout the wall. Uchida and Aoki [3] have examined a Newtonian fluid flow through a semi-infinite circular pipe. An analytical long series solution is obtained for a viscous fluid flow through a pipe of radius varies with time was discussed by Bujurke and Jayaraman [4]. Majdalani et. al. [5] have considered a Newtonian fluid flow through an expanding / contracting channel with weakly permeable walls and obtained the solution through numerically and as well as analytically. Boutros et. al. [6] have studied an incompressible fluid flow in a circular pipe with injection or suction at the wall whose radius dependent on time, then plotted the results through both analytically and numerically. Si et. al. [7] analyzed the fluid flow in a rectangular porous channel with expanding or contacting walls under the influence of the magnetic field. Drawn an important conclusion about viscoelastic parameter by HAM for an incompressible flow of a viscoelastic fluid between two porous walls and its distance is varies with time by Xin-Hui et. al. [8]. Xinhui Si et. al. [9] explained with homotopy analysis method of various parameter on different flow characteristics and energy conservations for an incompressible laminar micropolar fluid through a porous semi-infinite channel with expanding or contracting walls. Jafaryar et. al. [10] analyzed the efficiency of optimal homotopy asymptotic method through some the physical parameter of an isothermal viscous fluid flow in a parallel channel whose walls are expanding / contracting with respective to the time. Chao Wang et. al. [11] obtained a numerical solution by using matlab solver for a laminar incompressible fluid flow over an expanding / contracting porous channel. Rahimi et. al. [12] obtained a HAM, HPM solutions and compare with numerical method also to analyzing for two dimensional incompressible fluid through which the walls are moving in perpendicular to the flow direction. A semi-analytical approach is employed for MHD flow of nanofluid through an expanding / contracting porous pipe under the influence of heat source / sink in presence chemical reaction considered by Srinivas et. al. [13]. Xinhui Si et. al. [14] have examined a micropolar fluid flow through a semi-infinite deforming porous pipe with large suction / injection throughout the walls. A homotopy series solution is obtained to analyze the influence of chemical reaction on an unsteady laminar flow viscous fluid through an expanding / contracting porous pipe with uniform suction throughout the walls is considered by Srinivas et.

al.[15]. Radhika et. al. [16] investigated with series solution for an unsteady incompressible fluid flow through an expanding / contracting circular pipe whose radius depending on time.

Stokes (1966) was the first person who developed the theory of couple stress which is a generalization of the classical viscous fluid theory. The main the consideration of couple stresses is dependent on size of grain which is not considered in classical theories. The fluids containing of rigid, randomly oriented particles in the viscous medium. The fluids consisting of rigid, randomly oriented particles suspended in a viscous medium, such as blood, lubricants containing small amount of polymer additive, electro-rheological fluids and synthetic fluids. Najeeb et. al. [17] considered couple stress fluid flow in a porous channel with expanding / contracting walls and also suction / injection throughout the walls consequently a series solution is obtained. Ramzan [18] has analyzed a steady incompressible laminar MHD flow of couple stress nanofluid over a stretching sheet. A new version of the BVPh 2.0 Mathematica package based on the Homotopy analysis method (HAM) is used to solve nonlinear boundary-value [20-24]. Farooq et.al. [19] have considered MHD flow of a viscous nanofluid over a fixed wedge and solved by BVPh2.0. A two dimensional squeezing flow unsteady mixed convective nanofluid through parallel disks have been analyzed by Syed et. al. [20] by HAM. A squeezing flow of micropolar fluid through porous disks was considered by Khan et.al. [21] and attain a solution by BVPh2.0 HAM package. Zhao et. al. [22] have studied a nanofluid flow through a parallel disk by choosing of second order velocity slip condition at the boundary.

In this article, we have conducted a theoretical investigation of the influence of thermal radiation on unsteady laminar incompressible electrically conducting and mixed convective couple stress fluid flow through expanding or contracting porous pipe and also suction / injection at the walls in presence of chemical reaction. The governing constitute relation are transformed into a system of ordinary differential equations by using similar variables and then solved with HAM based BVPh2.0 Mathematica package and consequently we presented the results in details through graphs with respect to the different pertain non dimensional parameters.

II. Mathematical modeling of the Problem

Consider an unsteady, laminar, incompressible of and MHD flow of couple stress fluid through permeable porous pipe of radius is *a(t)* with uniform expanding / contracting wall

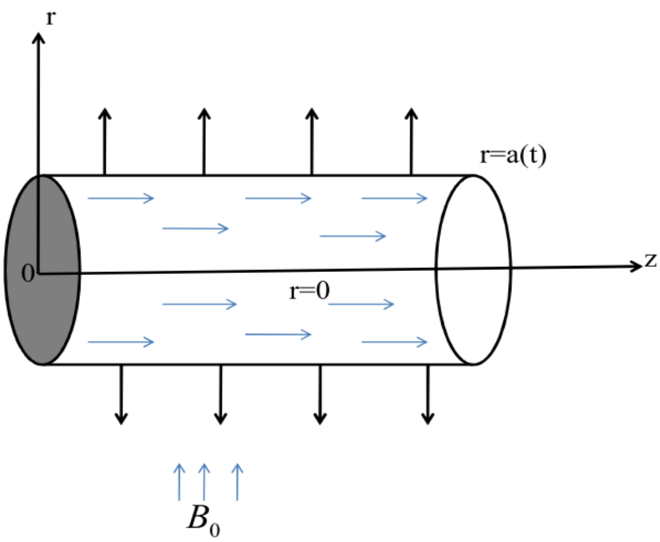


Fig. 1: Geometry of the system and bulk fluid motion.

in the normal direction flow with time. The fluid is sucked or injection uniformly and vertically through the wall with the absolute velocity v_w which is equal to the redial velocity of the fluid. The configuration of the flow system are shown though the cylindrical coordinates system (r,θ,z) in Fig.-1 the Z -axis is along flow direction, r -axis is along radial direction. We adopt a magnetic field of strength B_0 which is applied in the transverse direction and the velocity vector is $\bar{q}(w,0,u)$.

The conservation of mass, momentum, energy and concentration for an unsteady flow by considering the magnetic field, radiation and chemical reaction are,

$$u_z + w_r + \frac{w}{r} = 0$$

(1)

$$\rho\left[u_t + uu_z + wu_r\right] = -p_z + \mu\left[u_{zz} + u_{rr} + \frac{1}{r}u_r\right] - \eta\left[u_{rrrr} + \frac{2}{r}u_{rrr} - \frac{1}{r^2}u_{rr} + \frac{1}{r^3}u_r + 2u_{rrzz} + \frac{2}{r}u_{rzz} + u_{\partial rrzz} + u_{zzzz}\right] - \frac{\mu}{k_1}u - \sigma B_0^2 u + \rho g\beta_T\left(T - T_1\right) + \rho g\beta_C\left(C - C_1\right)$$

(2)

$$\rho\left[w_t + uw_z + ww_r\right] = -\frac{\partial p}{\partial r} + \mu\left[w_{zz} + w_{rr} + \frac{1}{r}w_r - \frac{w}{r^2}\right] - \eta\left[w_{rrrr} + \frac{2}{r}w_{rrr} - \frac{3}{r^2}w_{rr} + \frac{3}{r^3}w_r - \frac{3w^2}{r^4}u_{rrzz} - \frac{2}{r^2}u_{zz} + \frac{2}{r}u_{rzz} + u_{zzzz}\right] - \frac{\mu}{k_1}w - \sigma B_0^2 w$$

(3)

$$\rho c\left[T_t + uT_z + wT_r\right] = k\left[T_{zz} + T_{rr} + \frac{1}{r}T_r\right] + \mu\left[2\frac{u^2}{r^2} + 2\left(w_z\right)^2 + 2\left(u_r\right)^2 + \left(u_z + w_r\right)^2\right] + \eta\left[\left(u_{rz} - w_{rr}\right)^2 - \left(u_{zz} - w_{rz}\right)^2\right] + \sigma B_0^2\left(u^2 + w^2\right) + \frac{\mu}{k_1}\left(u^2 + w^2\right) - \left(q_r\right)_r \quad (4)$$

$$C_t + uC_z + wC_r = D\left[C_{zz} + C_{rr} + \frac{1}{r}C_r\right] - k_2\left(C - C_1\right)$$

(5)

Relevent boundary conditions are given by,

$$\left.\begin{array}{llllll} u = 0, & w = -v_w = -A\dot{a}, & \nabla\times\bar{q} = 0, & T = T_w, & C = C_w & at\ r = a(t) \\ \dfrac{\partial u}{\partial r} = 0, & w = 0, & \nabla\times\bar{q} = 0, & \dfrac{\partial T}{\partial r} = 0, & \dfrac{\partial C}{\partial r} = 0, & at\ \ r = 0 \end{array}\right\}$$

(6)

Where T_w, C_w are the temperature and concentration at wall respectively. $\dot{a} = da/dt$ is time dependent rate which is a velocity of the wall and $A = v_w / \dot{a}$ is a wall permeability measurement which is a constant. Introduced a suitable similar transformation in both space and time are given by [3, 5, 9, 11, 13],

$$u = \frac{\upsilon z F_\lambda\left(\lambda, t\right)}{a^2\lambda},\ \ v = -\frac{\upsilon F\left(\lambda, t\right)}{a\lambda},\ \ T = T_1 + \left(T_w - T_1\right)\phi\left(\lambda\right),\ \ C = C_1 + \left(C_w - C_1\right)\theta\left(\lambda\right)$$

(7)

where $\lambda = r/a\left(t\right)$ which is a dimensionless radial coordinate, T_1, C_1 are the reference temperature and concentration at the center of the pipe.

To analyzed the fundamental properties of the flow, preserves the nonlinear characteristics of the problem we attain a similar solution with respect to both space and time for which we consider the non-dimensional parameter α to be constant parameter and assume that $F_{\lambda\lambda t} = F_{\lambda t} = 0$ which means $F = R\ f\left(\lambda\right)$ [3],
By substituting the above similar transformation variables (7) and eliminating the pressure in momentum Eq. (2) - (5), then we get the following couple equations,

$$\beta^2\left(\lambda^5 f^{VI}-3\lambda^4 f^{V}+9\lambda^3 f^{IV}-24\lambda^2 f^{III}+45\lambda f^{II}-45 f^{I}\right)-\alpha\left(\lambda^6 f^{III}+\lambda^5 f^{II}-\lambda^4 f^{I}\right)+$$
$$R\left(\lambda^4 f^{I} f^{II}-\lambda^3 f^{I^2}-\lambda^4 ff^{III}+3\lambda^3 ff^{II}-3\lambda^2 ff^{I}\right)-\lambda^5 f^{IV}+2\lambda^4 f^{III}-3\lambda^3 f^{II}+3\lambda^2 f^{I}+$$
$$\lambda^4\left(D^{-1}+Ha^2\right)\left(\lambda f^{II}-f^{I}\right)-\left(\lambda^6/R\xi\right)\left(Gr\phi^{I}+Gc\theta^{I}\right)=0$$
(8)

$$\lambda^6\left(1+Rd\right)\phi^{II}+\lambda^5\phi^{I}+\Pr\lambda^5\left(\lambda^2\alpha+Rf\right)\phi^{I}+\Pr EcR^2\left\{2\lambda^2\xi^2\left(2f^{I^2}-2\lambda f^{II}f^{I}+\lambda^2 f^{II^2}\right)+\right.$$
$$\left.\lambda^4\left(D^{-1}+Ha^2\right)\left(\xi^2 f^{I^2}+f^2\right)+\beta^2\left(4\lambda^2 f^{II^2}+6\lambda^2 f^{I^2}+4f^2-12\lambda^3 f^{II}f^{I}-12\lambda f^{I}f+8\lambda^2 f^{II}f\right)\right\}=0$$

(9)

$$Sc\left(\lambda\theta^{II}+\theta^{I}\right)-\left(\lambda^2\alpha-Rf\right)\theta^{I}-\lambda K\theta=0$$
(10)

Where prime denotes the differentiation with respect to λ.The permeation Reynolds number defined by $R=\frac{av_w}{\upsilon}=A\alpha>0$ for injection and $R<0$ for suction. The non-dimensional wall expansion ratio define as $\alpha(t)=a\dot{a}/\upsilon$ is positive for expansion and negative for contraction.

In that event the wall expansion ratio $\alpha=\frac{a\dot{a}}{\upsilon}=\frac{a_0\dot{a}_0}{\upsilon}=\text{constant}\ (or)\ \frac{\dot{a}_0}{\dot{a}}=\frac{a}{a_0}$ by integrating with respect to the time and obtain a temporal similarity transformation which is given by [3],

$$\frac{a}{a_0}=\sqrt{1+2\upsilon\alpha t a_0^{-2}}$$
(11)

This implies the variation of the injection velocity is expressed as,

$$\frac{a}{a_0}=\frac{v_w(0)}{v_w(t)}=\sqrt{1+2\upsilon\alpha t a_0^{-2}}\ ,\quad \text{where}\quad v_w=A\dot{a}$$
(12)

Here a_0 is the initial radius value and $\dot{a}_0$ is the expansion rate.
The non-dimensional form of boundary conditions are given by

$$f^{/}(1)=0,\ \ f(1)=1,\ \ f^{//}(1)=0,\ \ \phi(1)=1,\ \ \theta(1)=1$$
$$f^{/}(0)=0,\ \ f(0)=0,\ \ f^{//}(0)=0,\ \ \phi^{/}(0)=0,\ \ \theta^{/}(0)=0$$
(13)

From eq-(7) dimensionless form of the temperature and concentration are given by,

$$\phi=\frac{T-T_1}{T_w-T_1}\quad and\quad \theta=\frac{C-C_1}{C_w-C_1}$$
(14)

From eqs. (2) and (3) the radial pressure gradient is given by,

$$p_\lambda=-\left(\alpha R^{-1}f+\frac{1}{2}\left(\frac{f^2}{\lambda^2}\right)+R^{-1}\left(\frac{f^{/}}{\lambda}\right)+R^{-1}\beta^2\left(\frac{f^{///}}{\lambda}-\frac{f^{//}}{\lambda^2}+\frac{f^{/}}{\lambda^3}\right)\right)^{/}$$
(15)

integrating eq (19) and centerline pressure p_c is given by,

$$\int_{p_c}^{p(\lambda)} dp = -\int_0^{\lambda}\left[\alpha R^{-1} f + \frac{1}{2}\left(\frac{f^2}{\lambda^2}\right) + R^{-1}\left(\frac{f^{/}}{\lambda}\right) + R^{-1}\beta^2\left(\frac{f^{///}}{\lambda} - \frac{f^{//}}{\lambda^2} + \frac{f^{/}}{\lambda^3}\right)\right]^{/} d\lambda$$

(16)

Therefore, the resulting pressure distribution will be

$$\Delta P_r = p(\lambda) - p_c$$

$$= R^{-1}\left[\frac{f^{/}}{\lambda} + \beta^2\left(\frac{f^{///}}{\lambda} + \frac{f^{/}}{\lambda^3}\right)\right]_{\lambda=0} - \left[R^{-1}\alpha f + \frac{1}{2}\left(\frac{f^2}{\lambda^2}\right) + R^{-1}\left(\frac{f^{/}}{\lambda}\right) + R^{-1}\beta^2\left(\frac{f^{///}}{\lambda} - \frac{f^{//}}{\lambda^2} + \frac{f^{/}}{\lambda^3}\right)\right]$$

(17)

Further, the shear stress can written as $\hat{\tau}_{rz} = \mu u_r + \frac{1}{4}\left[\lambda u_{rrr} - \frac{\lambda}{r} u_{rr} + \frac{\lambda}{r^2} u_r\right]$

(18)

Introducing dimensionless shear stress $\tau = \frac{\hat{\tau}}{\rho v_w^2}$ then the shear stress at the pipe wall is $\tau = \beta R^{-1}\frac{z}{a}\left[\frac{f^{IV}}{4} - f^{III}\right]$.

(19)

The heat (Nusselt number) and mass (Sherwood number) transfer rates are defined as

$Nu = -\frac{z}{a}\phi^{I}(\lambda)$, $Sh = -\frac{z}{a}\theta^{I}(\lambda)$ respectively.

Hence the dimensionless form Nusselt number and Sherwood number at the pipe wall are, given by, $Nu = -\phi^{I}(1)$, $Sh = -\theta^{I}(1)$ respectively.

(20)

III. Semi Analytical Solution of the Problem

A Mathematica package BVPh (new version) a free user friendly software on the bases of homotopy analysis method (HAM) for the problems of non-linear boundary- value and eigen value which was developed by Shijun Liao [23, 24]. It's possible to get a multiple solutions of highly non-linear boundary value problem for an finite / infinite intervals and singularities of governing equations with multi-point boundary conditions. Unlike it's also work for system of coupled non-linear ordinary differential equation and also provides a guaranteed convergence series solution (So called convergence control parameter) moreover, HAM gives access to give a large freedom of choosing initial approximation. So refer to the user's guide of the BVPh 2.0 online (http://numerical-tank.sjtu.edu.cn/BVPh.htm)

To solve equation (8) to (10) in view of that choosing initial guess which satisfied the non-dimensional boundary conditions eq (13) for $f(\lambda), \phi(\lambda)$ and $\theta(\lambda)$ as follows,

$f_0(\lambda) = 6\lambda^5 - 15\lambda^4 + 10\lambda^3$, $\phi_0(\lambda) = \lambda^2$ and $\theta_0(\lambda) = \lambda^2$.

Considering the auxiliary linear operators such the to meet highest order of the non-dimensional equations as follows,

$$L_1(f) = \frac{d^6 f}{d\lambda^6},\ L_2(\phi) = \frac{d^2\phi}{d\lambda^2} \text{ and } L_3(\theta) = \frac{d^2\theta}{d\lambda^2}$$

(21)

With the property

$$L_1\left(C_1\lambda^5 + C_2\lambda^4 + C_3\lambda^3 + C_4\lambda^2 + C_5\lambda + C_6\right) = 0,\ L_2\left(C_7\lambda + C_8\right) = 0,\ L_3\left(C_9\lambda + C_{10}\right) = 0$$

where $C_1, C_2, C_3, C_4, C_5, C_6, C_7, C_8, C_9$ and C_{10} are the constants.

The most important highlight that $f(\lambda),\phi(\lambda)$ and $\theta(\lambda)$ given by the package BVPh 2.0 containing three unknown convergence control- parameter C_0^f, C_0^ϕ and C_0^θ, which are used to control on convergence of the series solution.

The average residual error at the kth – order of approximation is used to reduced the CPU time, which is defined as,

$$\varepsilon_k^f\left(C_0^f, C_0^\phi, C_0^\theta\right) = \frac{1}{N+1}\left[N_f\left(\sum_{i=0}^{k} f_i, \sum_{i=0}^{k}\phi_i, \sum_{i=0}^{k}\theta_i\right)_{\eta=j\delta\eta}\right]^2$$

(22)

$$\varepsilon_k^\phi\left(C_0^f, C_0^\phi, C_0^\theta\right) = \frac{1}{N+1}\left[N_f\left(\sum_{i=0}^{k} f_i, \sum_{i=0}^{k}\phi_i, \sum_{i=0}^{k}\theta_i\right)_{\eta=j\delta\eta}\right]^2 \quad (23)$$

$$\varepsilon_k^\theta\left(C_0^f, C_0^\phi, C_0^\theta\right) = \frac{1}{N+1}\left[N_f\left(\sum_{i=0}^{k} f_i, \sum_{i=0}^{k}\phi_i, \sum_{i=0}^{k}\theta_i\right)_{\eta=j\delta\eta}\right]^2 \quad (24)$$

Where N is an integer and total error at the kth-order of approximation is given by,

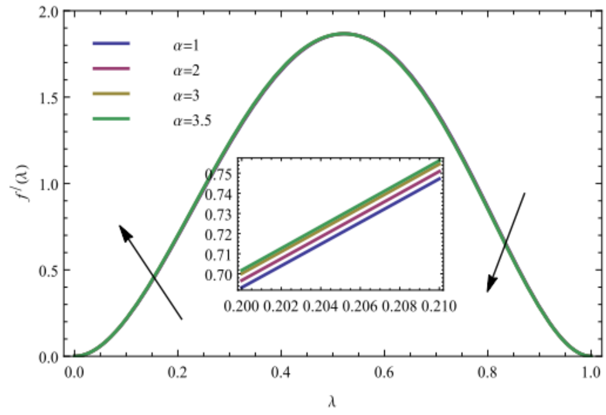
$$\varepsilon_k^t\left(C_0^f, C_0^\phi, C_0^\theta\right) = \varepsilon_k^f\left(C_0^f, C_0^\phi, C_0^\theta\right) + \varepsilon_k^\phi\left(C_0^f, C_0^\phi, C_0^\theta\right) + \varepsilon_k^\theta\left(C_0^f, C_0^\phi, C_0^\theta\right)$$

(25)

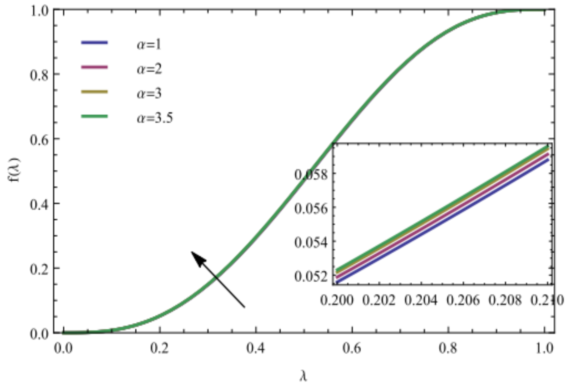
Therefore by optiminizating the value of C_0^f, C_0^ϕ and C_0^θ we minimizing the total error ε_0^t 10th -order approximation, which can be done by simply using "GetOptiVar", a command of the BVPh2.0.

IV. RESULTS AND DISCUSSION

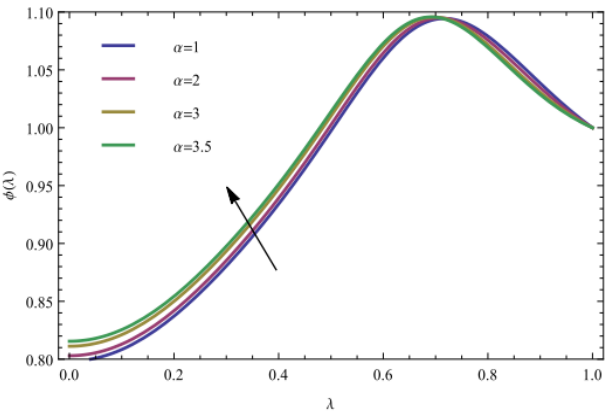
This section is dedicated to present the impact of various emerging physical non-dimensional parameters such as wall expansion ratio (α), Couple stress parameter(β), Hartmann number (Ha), Reynold's number (R), inverse Darcy number $\left(D^{-1}\right)$, chemical reaction (K) and thermal radiation (Rd) on dimensionless velocities, temperature distribution, concentration profile. Fig.2 describes the impact of α on velocity profiles, temperature and concentration distributions. As α increases the redial velocity, temperature are also increases where as the concentration is decreases however, the axial velocity is attain maximum at centreline of the channel then decreases. The Fig. 3(a)-3(d) revels the effects for different values of couple stress parameter β with respective velocity profile, temperature and concentration. The radial velocity profile $f(\lambda)$ and temperature $\phi(\lambda)$ are increasing with increasing of couple stress parameter (β) whereas the concentration $\theta(\lambda)$ is decreasing however, the axial velocity is increases $f^{/}(\lambda)$ towards the central of the channel. It means that when the couple stresses are maximum, the transverse velocity and temperature of the fluid is increased whereas the concentration of the fluid decreases this is due to the fact that the couple stress parameter inversely proportional to the viscosity and radius of the pipe. From Figs. 4(a) to 4(d) display the impact of Hartmann number (Ha) on non-dimensional flow characteristic profiles, thermal and concentration diffusions. It is notified that when Ha increases the temperature is also increases whereas, radial velocity and concentration are decreasing towards the wall λ=1 However, axial velocity decreases in the first half then increases since the Lorentz force retarded the flow. Figs. 5(a)-5(d) presence the non-dimensional velocity, temperature and concentration profiles for different value permeation Reynolds number (R). These show that as R increases the non-dimensional temperature profile and concentration profile are increasing and the reverse trend has seen for radial velocity. However, the axial velocity decreases towards the centreline of the channel and then increases towards the upper wall $\lambda=1$. The effect of inverse Darcy parameter (D^{-1}) on non-dimensional functions $f^{/}, f, \phi$ and θ against λ are reported in the Fig. 6(a) - 6(d). It interprets that the effect of D^{-1} which is similar trend of Ha for axial and radial velocity components, thermal diffusivity and the concentration distribution of the fluid decreases with increasing of D^{-1}. Fig. 7(a)-7(d) display that the impact of chemical reaction parameter on non-dimensional velocities functions, temperature and concentration. It shows that radial velocity, temperature and concentration are increases with increasing of K whereas the axial velocity is increasing initially up to the middle of the channel then decreases towards upper plate. Fig. 8(a)-8(d) revels that the influence of the non-dimensional functions $f^{/}, f, \phi$ and θ against λ for various values of the thermal radiation (Rd). Its represents the effect of Rd which is follows the similar trend with couple stress parameter as well as wall expansion ratio.



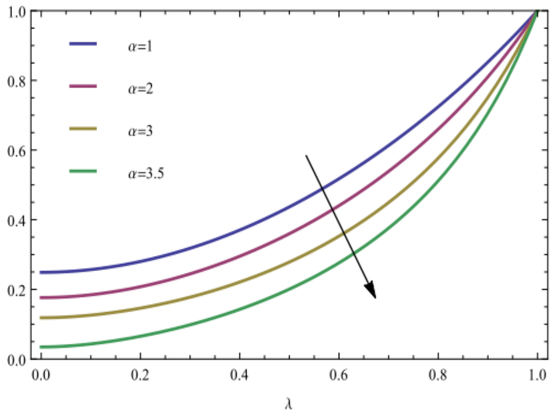
2(a)



2(b)

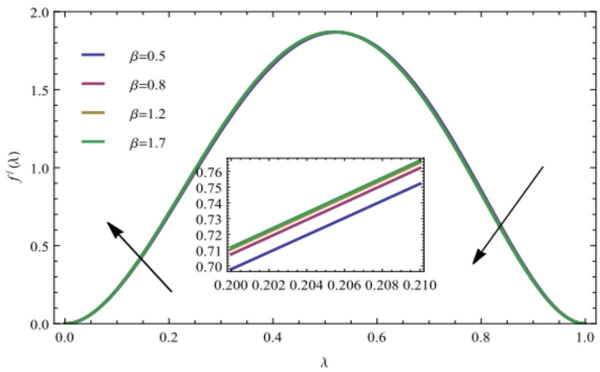


2(c)

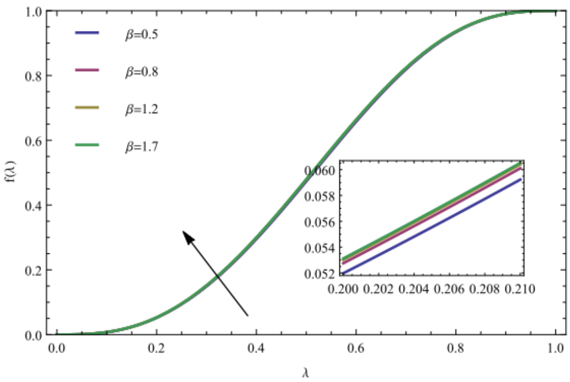


2(d)

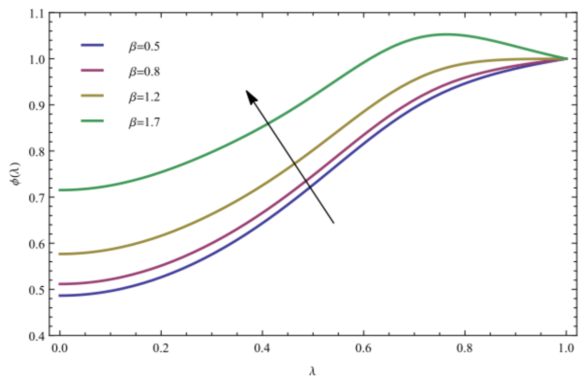
Fig. 2: Effect of α on (a) Axial velocity, (b) Radial velocity, (c) Temperature and (d) Concentration β = 0.5, R = 0.2, D^{-1} = 10, Ha= 0.5, Pr = 0.71, Rd = 0.5, Gr = 0.04, Gc = 0.04, Sc = 0.7, Ec = 0.05, ξ = 0.4, k = 0.5.



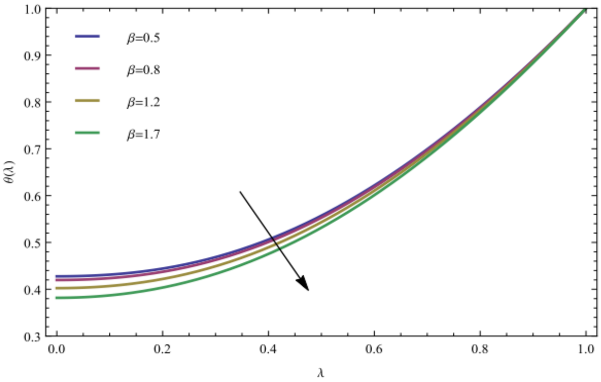
3(a)



3(b)

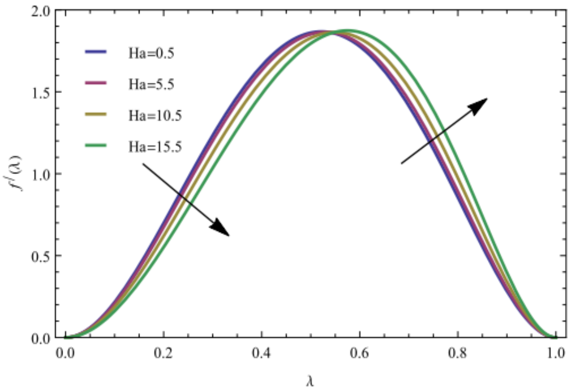


3(c)

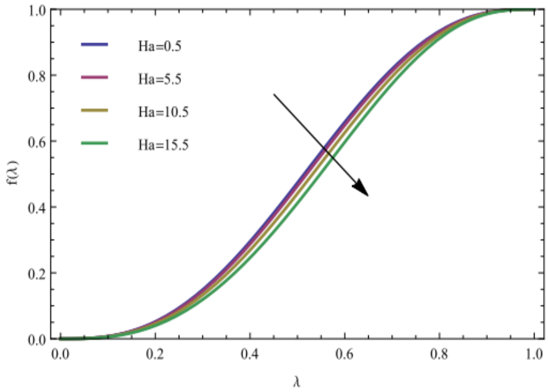


3(d)

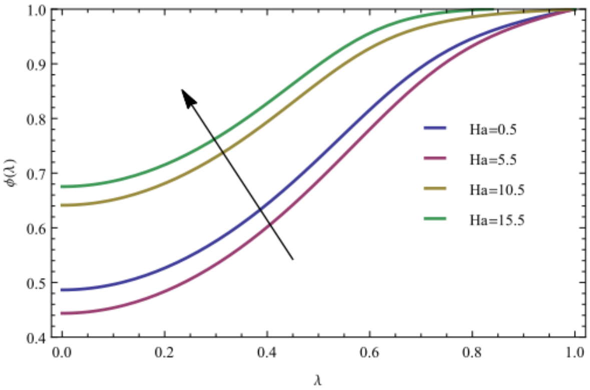
Fig. 3: Effect of 'β' on (a) Axial velocity, (b) Radial velocity, (c) Temperature and (d) Concentration α = 0.5, R = 0.2, D^{-1}= 2, Ha= 0.5, Pr = 0.71, Rd = 0.5, Gr = 0.04, Gc = 0.04, Sc = 0.7, Ec = 0.05, ξ = 0.4, k = 0.5.



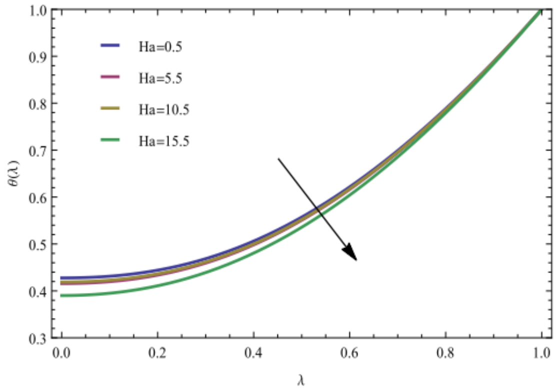
4(a)



4(b)

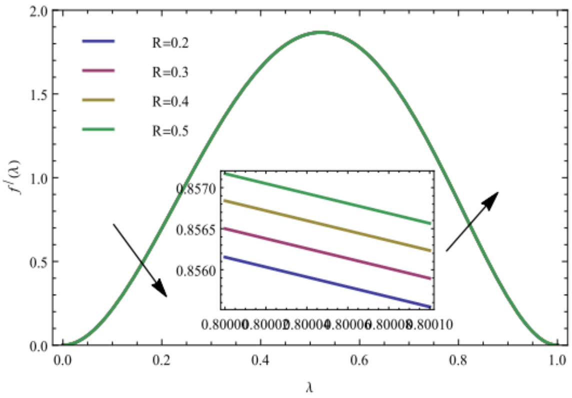


4(c)

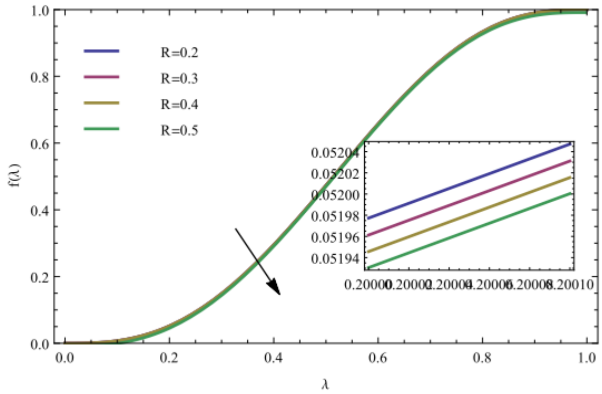


4(d)

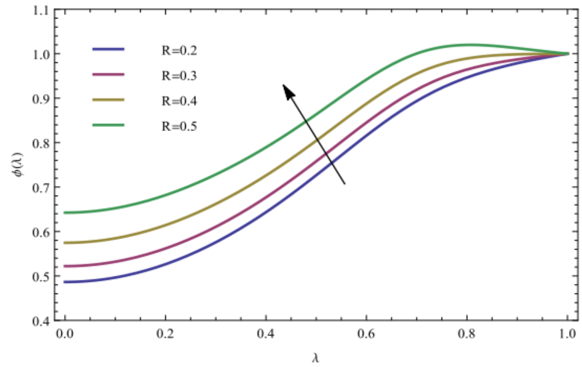
Fig. 4: Effect of 'Ha' on (a) Axial velocity, (b) Radial velocity, (c) Temperature and (d) Concentration β = 0.5, α = 0.5, R = 0.2, D^{-1}= 2, Pr = 0.71, Rd = 0.5, Gr = 0.04, Gc = 0.04, Sc = 0.7, Ec = 0.05, ξ = 0.4, k = 0.5.



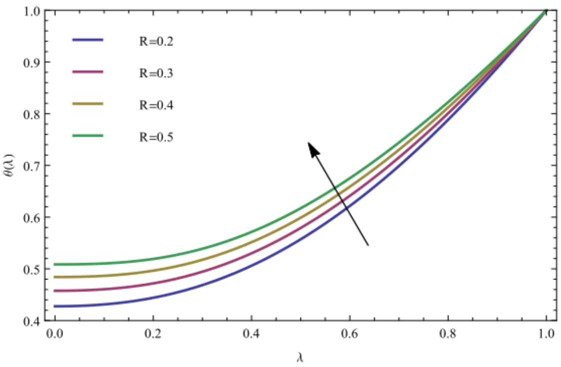
5(a)



5 (b)

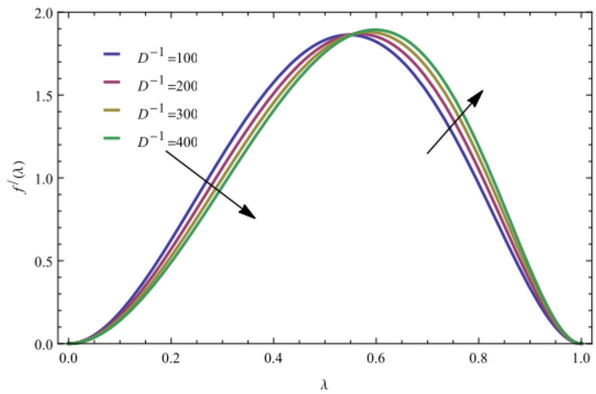


5(c)

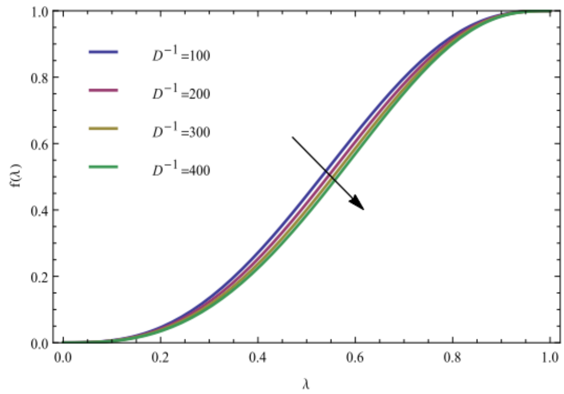


5(d)

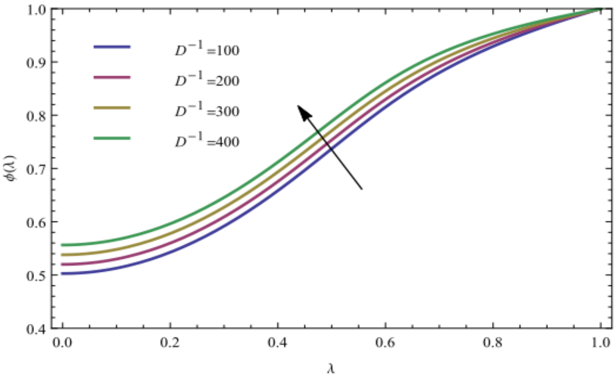
Fig. 5: Effect of R on (a) Axial velocity, (b) Radial velocity, (c) Temperature and (d) Concentration β = 0.5, α = 0.5, R = 0.2, D^{-1}= 2, Ha = 0.5, Pr = 0.71, Rd = 0.5, Gr = 0.04, Gc = 0.04, Sc = 0.7, Ec = 0.05, ξ = 0.4, k = 0.5.



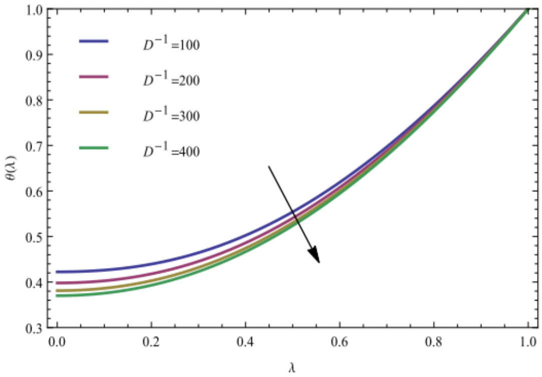
6(a)



6 (b)

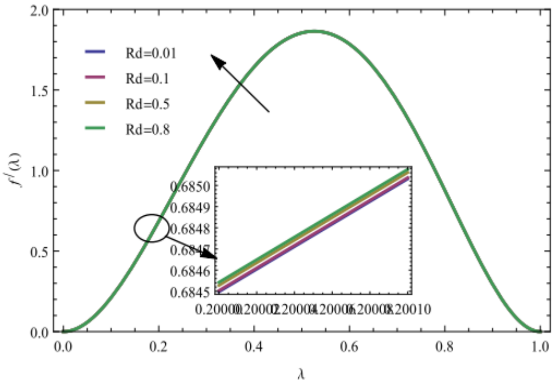


6(c)

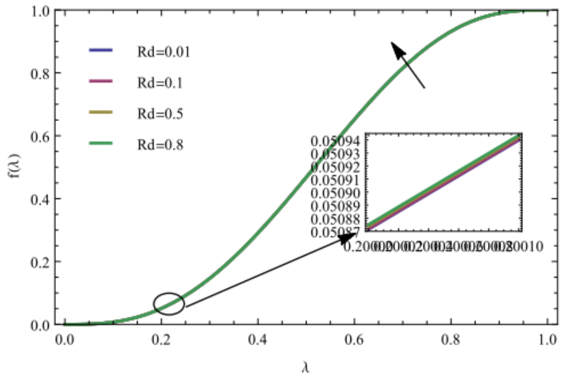


6(d)

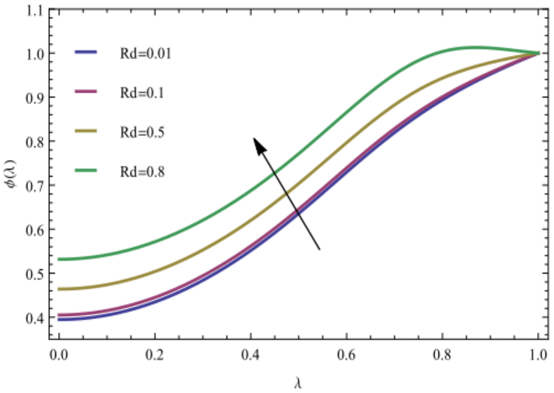
Fig. 6: Effect of R on (a) Axial velocity, (b) Radial velocity, (c) Temperature and (d) Concentration β = 0.5, α = 0.5, R = 0.2, D^{-1}= 2, Ha = 0.5, Pr = 0.71, Rd = 0.5, Gr = 0.04, Gc = 0.04, Sc = 0.7, Ec = 0.05, ξ = 0.4, k = 0.5.



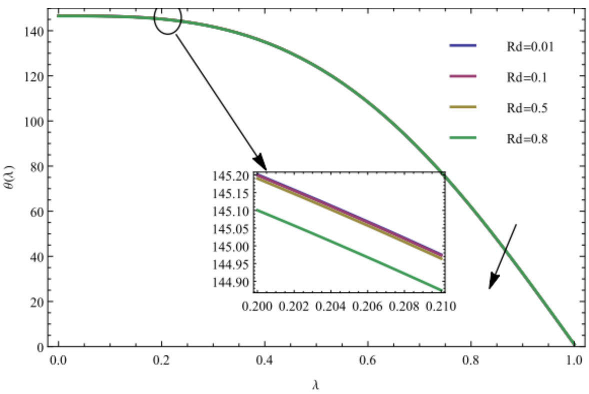
7(a)



7 (b)

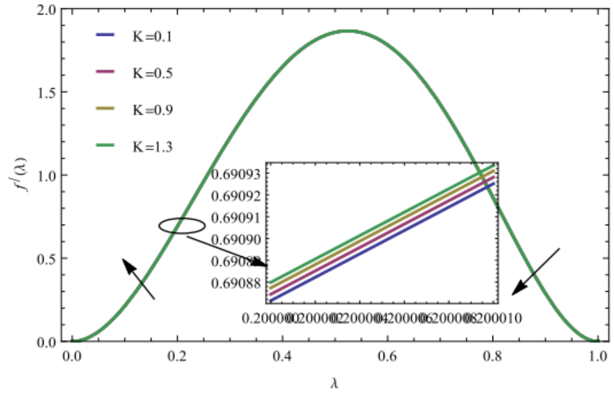


7(c)

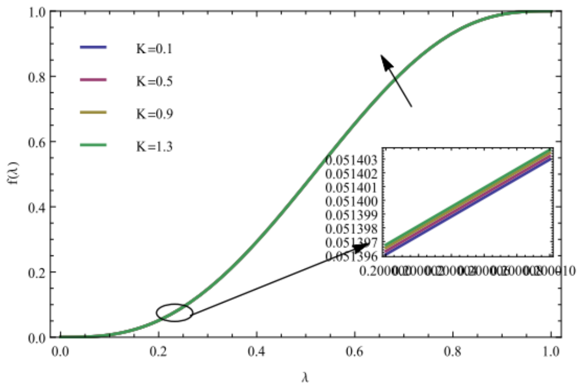


7(d)

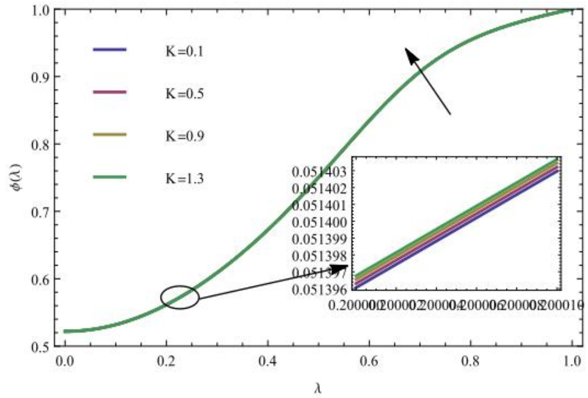
Fig.7: Effect of Rd on (a) Axial velocity, (b) Radial velocity, (c) Temperature and (d) Concentration β = 0.5, α = 0.5, R = 0.3, D^{-1}= 10, Ha = 0.5, Pr = 0.71, Gr = 0.09, Gc = 0.09, Sc = 0.1, Ec = 0.05, ξ = 0.4, K = 0.05.



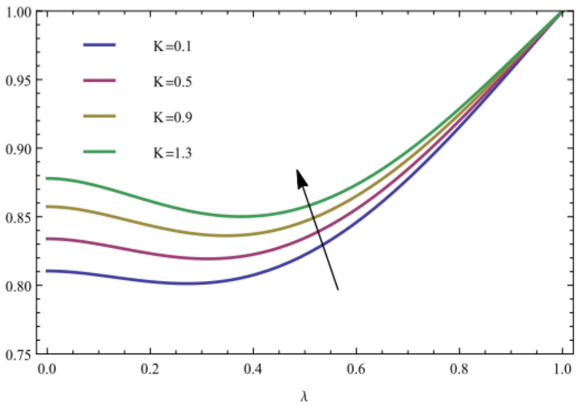
8(a)



8(b)



8(c)



8(d)

Fig. 8: Effect of K on (a) Axial velocity, (b) Radial velocity, (c) Temperature and (d) Concentration β = 0.5, α = 0.5, R = 0.5, D^{-1}= 10, Ha = 0.5, Pr = 0.71, Rd = 0.05, Gr = 0.25, Gc = 0.25, Sc = 1.3, Ec = 0.05, ξ = 0.4.

V. CONCLUSIONS

In this article we considered an unsteady incompressible laminar MHD flow of mixed convective and chemically reactive couple stress fluid through expanding or contracting porous pipe in presence of thermal radiation is employed. The constitute equations that governing flow configuration are reduced to nonlinear coupled ordinary differential equations by using appropriate similar variables and obtained solution with HAM based Mathematica package BVPh2.0. From the results we are conclude that

- The velocity of the fluid is increased by wall expansion/ contraction ratio whereas the concentration is decreased.
- Temperature and concentration of the fluid are in opposite nature for increasing of Rd.
- The concentration of the fluid is decreased with increasing of Ha and D^{-1}.
- The velocity profiles and temperature distribution of the fluid are opposite to each other for Ha and D^{-1}.

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